Compactness of powers of strictly singular operators

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1.- Some properties of strictly singular operators

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2.-Extensiones de un resultado de Milman a Retículos de Banach

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- with *J. Flores(URJC), E.Semenov(VSU) and P. Tradacete (UB)*

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$\mathcal{DP}$ Dunford-Pettis oper.; $\mathcal{WK}$ Weakly compact oper.

$$\mathcal{WK} \cap \mathcal{DP} \subset \mathcal{SS}$$

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Even *super* strictly singular operators fail it (*Chalendar, Troitsky,... 2009)*

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- case $2 < p < \infty$ use Kadec-Pelcynski disjointification met.
(If (x_n) seminorm. basic s. $\Rightarrow$ there is $(x_{n_k}) \sim (e_k)$ of ℓ_p or ℓ_2)

(x_n) is weak convergent $\Rightarrow$ there is (x_{n_k}) s.t. $(T^2 x_{n_k})$ converges .

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- case $1 < p < 2$ use duality, Schauder T., Lp' subprojective

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-3.- When does exist $T \in SS(E)$ s.t. $T^n \notin K(E)$ for every n ? (i.e. T non-power compact)

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Let $1 \leq r \neq s \leq \infty$ *and* $T : L_s \to L_s$ *bounded .*
If $T \in SS(L_r) \implies T \in K(L_p)$ *for every* $p \in (r, s)$

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Flores, Tradacete, Troitsky (2009) - Compact product of regular operators on Banach lattices

• E **disjointly homogeneous (DH)** if two disjoint normaly. (x_n), (y_m) have subsequuences $(n_k), (m_k)$ s. t.

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$L_{p,1}$ is 1-DH but $(L_{p,1})^*$ is not DH

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DH Banach lattice E with finite cotype and unconditional basis.

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$1 < p < 2$:

$$\begin{array}{ccc}
L_{p,\infty} & \xrightarrow{\quad T \quad} & L_{p,\infty} \\
\Big\downarrow P & & \Big\uparrow J \\
\ell_p \oplus \ell_{p,\infty} \oplus \ell_2 \oplus \ell_\infty & \xrightarrow{\quad S \quad} & \ell_p \oplus \ell_{p,\infty} \oplus \ell_2 \oplus \ell_\infty \\
(\,x\ ,\ y\ ,\ w\ ,\ z\,) & \longmapsto & (\,0\ ,\ x\ ,\ y\ ,\ w\,)
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$T \in \mathcal{SS}(L_\varphi) \Rightarrow T^2 \in \mathcal{K}(L_\varphi)$

For $p = 2$ $\quad \mathcal{SS}(L_\varphi) = \mathcal{K}(L_\varphi)$

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An *Orlicz* L_φ with $s_\varphi = \sigma_\varphi$, and $T \in \mathcal{SS}(L_\varphi)$ st $T^2 \notin \mathcal{K}(L_\varphi)$.

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L_\varphi \oplus L_\varphi \oplus L_\varphi & \xrightarrow{T} & L_\varphi \oplus L_\varphi \oplus L_\varphi \\
P_R \downarrow \quad P_\varphi \downarrow & & \uparrow J_\varphi \quad \uparrow J_p \\
\ell_2 \oplus \ell_\varphi & \xrightarrow{S} & \ell_\varphi \oplus \ell_p
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T is power compact $\Longleftrightarrow$ for every bounded $(x_n) \subset E$ there is a $p = p((x_n)) \in \mathbb{N}$ such that $(T^p x_n)$ has a convergent subsequence

Tacon 1979, Brown-Foias 1981

Theorem

$1 \leq p < q < \infty$, *there exist Orlicz* L_φ *st* $s_\varphi = p < \sigma_\varphi = q$ *and* $T \in \mathcal{SS}(L_\varphi)$ *non-power compact.*

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$1 \leq s_E \leq p_E \leq q_E \leq \sigma_E \leq \infty$

(X_1, X_2) interpolation pair. For $z \in X_1 + X_2$

$$k(z, a, b) := \inf\{a\|x_1\|_{X_1} + b\|x_2\|_{X_2} : z = x_1 + x_2\}.$$

Y with unconditional b. (y_n) , (a_n), (b_n) ,, $\sum_{n=1}^{\infty} \text{mín}(a_n, b_n) < \infty$.

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$\varphi(t) = t^p \log(1 + t)$ $(1 < p < \infty)$

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Tsirelson type spaces T_θ^p . For $x = (x_n) \in c_{00}$, and $A \subset \mathbb{N}$, $A(x) = (x_n)_{n \in A}$,for $0 < \theta < 1$,

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